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CONCEPT EVALUATION FOR OCEAN SHEAR PROFILER.(U)  
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## CONCEPT EVALUATION FOR OCEAN SHEAR PROFILER

### Introduction

There presently exist several R&D devices that provide ocean shear profile information, either in a moored or a free-fall mode of operation. It would be exceedingly useful to have a device that could provide shear profile information in the over-the-side mode commonly used for acquiring ocean station hydrographic data. Present instruments that are operated in this mode suffer severe degradation of performance due to ship motions. The purpose of this note is to describe a new concept that should be useful for this purpose but which still requires a thorough engineering design study and prototype construction, test, and evaluation to be certain that instrument noise criteria can be met.

### Present Technology

The spatial and temporal variability in upper ocean physical, chemical, and biological parameters is governed by dynamical processes that occur on many space and time scales. The unraveling of these processes requires detailed information on the fluxes of kinetic and potential energy in the upper ocean layers. To meet these information requirements, new technology has provided the oceanographer with sophisticated instruments that measure water temperature, conductivity and velocity, as well as other less dynamically important parameters. In the last decade, improvements in instrumentation have provided the CTD profilers that presently yield accurate conductivity and temperature profiles on a survey basis. However, the accurate measurement of water velocity profiles is only presently being attempted with specialized research

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instruments. There are several types of these profilers that have shown promise, but they are not in routine use on survey vessels. They can roughly be grouped into mooring, free-fall and underway types.

The tethered cyclesonde of VanLeer et al (1974) is a current meter that has a controllable buoyancy element and is attached by a sliding connector to a mooring cable. At a controllable rate, it slides up and down the cable, yielding repeated profiles at the mooring location. However, the mooring float is constrained to be well below surface effects (cf. Gould, Schmitz and Wunsch, 1974) to prevent contamination of the velocity records by mooring motions, so uncontaminated near-surface profiles are not attainable.

The free-fall types are launched from a research vessel, they descend to a prescribed depth or the bottom, ascend again, and are recovered. The purpose of having them untethered is to prevent contamination by ship or mooring motions, but of course station time and ease of handling suffer appreciably. There have been several types of profilers in this class. Sanford (1975) uses an EM device that measures the voltages induced by the movement of the sea and the instrument through the geomagnetic field. Rossby (1969) has acoustically tracked a freely falling dropsonde, and, as with Sanford's instrument, the vertical resolution is the order of 10 meters. Simpson (1975) has used a tiltmeter to deduce smaller scales down to order 30 cm, and Osborne (1974) has obtained vertical resolutions to 1 cm by measuring the force on a spinning cantilevered vane. These increased resolutions come only at the expense of information on the longer scales. For ease in handling, Sanford has been developing an expendable profiler which is based upon a combination of the technology in his EM profiler and in the XBT drop body and wire link. This of course has the advantage of being deployable while underway.

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Finally, underway profilers using acoustic Doppler backscatterometry are being developed which are modifications of commercially available bottom speed indicators for supertanker navigation. These profilers may be useful, but they potentially are very sensitive to surface wave induced ship motions and to the density of volume scatterers in the water column. They also practically are limited to the top few tens of meters of water, and quite possibly will not be useful in the present context.

There is a need for a profiler that can be deployed in the usual manner in which the CTD is deployed and which has comparable resolution. In fact, it should use the same hydrographic cable so that both the density and velocity profiles can be acquired at the same time. It is the purpose of this note to identify what appears to be a workable system that is relatively immune to ship motions.

## **Ship Motions and Current Meters**

The problem with deploying traditional current meters on hydrographic wire to obtain velocity profiles is sensor motion due to coupling with the ship. There are two types of motion that cause problems. First, any vertical motion such as raising, lowering, or ship's roll causes a vertical component of flow past the meter. In mechanical meters, this "pumps up" the energy level because of nonlinear coupling of the longitudinal, up/down motions to the near-horizontal motions in the ocean. This problem, in principle, could be eliminated by nonmechanical measurement systems such as the Neil Brown acoustic current meter (N.B.I.S., 1978). Second, drift of the ship drags the instrument horizontally through the water at a rate that can be much greater than the signal being measured. This causes an indicated velocity that may bear little resemblance to the real one, and it also tilts the instrument so that only a component of the horizontal velocity is registered.

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The effect of both of these motions can be eliminated by a judicious choice of the parameter being measured. In this instance, we will argue that the direct measurement of fluid vorticity yields a useful technique for measuring the required water velocity information while at the same time suppressing the noise caused by instrument motion.

### Theory

The instrument should be designed to measure the horizontal components of vorticity

$$\begin{aligned}\zeta_x &= \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \\ \zeta_y &= \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\end{aligned}\tag{1}$$

where (x,y) are horizontal axes, z is the vertical axis, and (u,v,w) are the two horizontal and the vertical component of velocity. The meter will be concerned primarily with horizontal velocities, since the vertical velocity is appreciable only in small scale motions. In fact, an approximation in which the vertical motions are entirely neglected is well grounded in the use of all previous current meters. However, since vorticity is to be measured, the relation between the vorticity and the vertical profile of horizontal current is worth reconsidering.

Low frequency (mesoscale and below) motions do not have appreciable vertical components, but the higher frequency motions may. In the important internal wave band, the vertical velocities can be appreciable. By continuity, the vertical velocity is related to the horizontal one by

$$w \sim k_x/k_z u,\tag{2}$$

where  $k_x$  and  $k_z$  are the horizontal and vertical wave numbers of a plane wave. By the dispersion relation,

$$k_z \sim k_x \left( \frac{N^2 - \omega^2}{\omega^2 - f^2} \right)^{1/2},\tag{3}$$



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where  $N$  is the local buoyancy or Vaisala frequency and  $f$  is the inertial frequency, so that

$$w/u \sim \left( \frac{\omega^2 - f^2}{N^2 - \omega^2} \right)^{1/2}. \quad (4)$$

Thus, we conclude that waves with frequencies near the local Vaisala frequency have vertical velocities as large as or larger than the horizontal velocities. However, there is not much energy concentrated near the Vaisala frequency.

The Garrett and Munk model spectrum of internal wave fluctuations (Garrett and Munk, 1975) implies that the internal wave energy falls off as  $\omega^{-2}$  in the frequency domain. With this spectrum, and for reasonable values of  $N$  and  $f$ , 90% of the energy due to the internal waves is in a frequency band an order of magnitude below the Vaisala frequency. In this band, the vertical velocities all are less than 10% of the horizontal velocities. Thus, we do not do violence to most of the internal wave band of motions.

Now, it is expected that  $\partial w / \partial x$  and  $\partial w / \partial y$  are negligible compared to vertical derivatives of horizontal velocities for mesoscale and lower frequency motions. For internal waves, the theory gives

$$\begin{aligned} \frac{\partial w / \partial x}{\partial u / \partial z} &\sim \frac{k_x w}{k_z u} \\ &\sim \frac{\omega^2 - f^2}{N^2 - \omega^2}. \end{aligned} \quad (5)$$

For measured frequency spectra, the numerator is two orders of magnitude less than the denominator in the lower frequency band in which 90% of the energy resides. This implies that the vertical shear is the dominant contributor to the vorticity in this band also. This approximation breaks down for the higher frequency waves for which the wave frequency is order of a third the buoyancy frequency or larger, and, in this band, measurements of vorticity are more

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useful than those of horizontal velocity. In principle, the ratio  $\zeta/N$  is a measure of the strength of these wave motions in the absence of fine structure.

For turbulent motions, either in the near-surface active mixed layer or in the microscale patches of active turbulence in the interior of the ocean, all terms on the right hand side of equations 1 are comparable in magnitude. Since the vorticity is a fundamental turbulence property, the measurement of it is more useful in understanding the dynamics than is a measurement of horizontal velocity. In addition, a direct measurement of vorticity near the ocean surface in a search for the turbulence levels, momentum fluxes, and the like in the vicinity of the surface has the advantage over a velocity measurement of being practically independent of surface wave motions. The orbital motions of surface waves are very nearly irrotational so that this instrument automatically suppresses them.

Finally, there is a further practical advantage of a vorticity measurement over that of velocity because of the shape of the wavenumber spectrum of motions. What limited information that is available on both the vertical and horizontal structure of motions in the ocean indicates that the spectra are decidedly red; that is, spectral estimates of the longer waves are considerably higher than those of the shorter waves. The spectral rolloff is the order of the square of the inverse wavenumber, so that the slope spectra of velocity would be more or less white. Since the vorticity involves spatial derivatives of velocities, it is expected that spectra of the vorticity also will be approximately white. Given a choice of which variable to measure, the vorticity enjoys a considerable advantage due to its expected white spectrum because this results in higher reliability by the avoidance of side-lobe effects.

In the assumption in which the last terms in equation 1 are neglected, the vertical profile of horizontal velocity simply is an integral of the vorticity profile. The velocity profile obtained

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from the integral of a measured vorticity profile is a relative one unless an accurate estimate of water velocity at some depth over the time of the cast is available. For a cast through the permanent thermocline that takes the order of an hour to complete, present satellite or Loran C navigation systems might be accurate enough to estimate ship drift for this purpose. For very shallow casts through only the mixed layer and seasonal thermocline, a more accurate system such as the future NAVSTAR Global Positioning System or a short range radio positioning system would be required.

#### Sensor Considerations

There are many types of instrument configurations that can sense water movement. Non-mechanical ones receive high priority because of their reliability and relative immunity to contamination. Acoustic devices, such as the new Neil Brown current meter (N.B.I.S., 1978), have high resolution, accuracy, and dynamic range, and they have lower thresholds. However, unless specially instrumented, they suffer from ship motion.

The new device considered here will be called a vorticity meter. Mechanical devices have previously been designed to measure fluid vorticity (cf. Todd, 1949; Hopkins & Sorensen, 1956; Bobrik, 1960; Lokshminarayana, 1964; Zalay, 1976; Wigeland et al, 1978; Govinda Raju et al, 1978), but these all are of the rotor or vane type. The large number of references on the vane-type meter is due to many attempts to construct responsive, calibrated instruments. This technique still involves considerable art in constructing the devices, and they undoubtedly would experience difficulties if used for any length of time in seawater. Nevertheless, Gongwer and Finkle (1962) have deployed a meter of this type at sea to assess the level of "turbulence" there. Although calibration and longevity are not discussed in their article, the instrument apparently did survive the sea test in working order. Kovasznay (1954) has discussed an

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arrangement of hot wire probes which measures fluid vorticity, but, even in a hot film instead of hot wire arrangement, these probes suffer calibration drift and high failure rates in seawater. An acoustic vorticity meter has been designed for a towed device (Gibson and Williams, 1978) and has been discussed in terms of ocean basin scale measurements (Rossby, 1975), so only its use in the profiling configuration can be considered to be entirely new. The instrument would be deployed in the same manner as a CTD and, in its most efficient mode, both devices would utilize the same pressure case and electronics. Two acoustic devices below, or on the side of, the case would measure the two horizontal components of vorticity. Each device would be a planar array of projector, reflectors, and receiver, that would measure the circulation around a closed acoustic path in the water. This measurement is accomplished by measuring the time difference that a pulse takes to travel opposite directions around the path, and relating this directly to the circulation around the path. This circulation  $\Gamma$  is defined as

$$\Gamma = \oint \mathbf{u} \cdot d\mathbf{l} \quad (6)$$

where  $\mathbf{u}$  is the velocity vector and  $d\mathbf{l}$  is the differential path length. By Stokes theorem,

$$\oint \mathbf{u} \cdot d\mathbf{l} = \int \boldsymbol{\zeta} \cdot \mathbf{n} dA \quad (7)$$

where  $\boldsymbol{\zeta} = \nabla \times \mathbf{u}$  is the vorticity vector and  $\mathbf{n}dA$  is an element of any surface bounded by the closed acoustic path. Assuming a planar array and uniform distribution of vorticity in the enclosed area, this reduces to

$$\Gamma = \zeta A, \quad (8)$$

where  $\zeta$  is the vorticity component perpendicular to the plane of the array and  $A$  is the total enclosed area of the array. Thus the vorticity is given by

$$\zeta = \Gamma/A, \quad (9)$$

where  $\Gamma$  is measured and  $A$  is given by the array configuration.

The travel time of an acoustic pulse (or constant phase point of a CW signal) is

$$T = \frac{L^2}{cL + \int_0^L \mathbf{u} \cdot d\mathbf{l}} \quad (10)$$

where  $L$  is the path length,  $c$  is the sound velocity and it has been assumed that the variation in sound speed around the array is small. The difference for propagation each way, then, is

$$\Delta T = \frac{2}{c^2} \int_0^L \mathbf{u} \cdot d\mathbf{l} \quad (11)$$

where it also has been assumed that  $cL \gg |\int_0^L \mathbf{u} \cdot d\mathbf{l}|$ . Note that the formula above does indicate that the local value of the sound speed may be important. With this, and the previous formula, the vorticity is given by

$$\zeta = \frac{c^2 \Delta T}{2A}. \quad (12)$$

Two such devices yield two components of horizontal vorticity, and, when referenced to an internal compass, yield  $\zeta_x$  and  $\zeta_y$ . Alternatively, a single array on a spinning device with an internal compass could provide the required information. (See the Appendix for more details in the derivation of Eq. 12.)

There are several critical design factors that must be addressed before such a system would be considered to be entirely feasible. First, the size of the array defines the resolving power of the instrument. The size would be a trade-off between the required resolution and the accuracy and repeatability of the clock and pulse generator, and preliminary estimates of this size can be made. The Richardson number defined by

$$R_i = N^2 / (du/dz)^2 \quad (13)$$

often is between 1 and 100 on scales of meters to tens of meters in the ocean. Taking the larger number as representative of a minimum value of vorticity to design for, and assuming a typical buoyancy frequency of  $10^{-2}$  rad/sec in the upper ocean, a typical noise floor of

$$\zeta \sim 10^{-3} \text{ rad/sec}$$

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seems reasonable. (Note that this could be an order of magnitude smaller in the deeper ocean.) Assuming a state-of-the-art timing accuracy of 100 picoseconds ( $10^{-10}$  sec) for oceanographic instrumentation (Neil Brown Acoustic Current Meter) and a sound speed of 1500 m/s, equation 12 allows a smallest acoustic array cross section of  $0.1 \text{ m}^2$ . This corresponds to a triangular array having legs of 50 cm. An order of magnitude increase in timing accuracy to 10 picoseconds would allow this to be reduced to about 15 cm on a side.

The second problem area is the level of contamination due to instrument motion. This design is entirely immune to translational motions, but it is susceptible to instrument rotations. Rotation of the acoustic array about an axis perpendicular to its own plane causes a signal that appears identical to fluid rotation due to vorticity in the form  $\zeta = 2 \, d\theta/dt$ , where  $d\theta/dt$  is the roll rate. This could be subtracted out by proper instrumentation such as a roll rate gyro since  $10^{-3}$  rad/sec is within that technology, but it incurs the penalty of extra cost and complexity. This motion is due to ship motion and to the effects of the shear on the instrument casing, and it can be modelled or tested experimentally in order to accurately assess its impact on the design. The crucial point is to optimize the design of the instrument so that the tilt effects are minimized.

A third problem is the calculation of a velocity profile from the vorticity profile. For many uses, this calculation is not necessary because it is either the vorticity or the vertical shear that actually is required. For example, the profile of Richardson number, defined by equation 13 is an important parameter in energy transfer processes. However, for those problems for which the velocity is required, the measured shear profile must be integrated in the vertical to obtain the velocity profile. The resulting velocity profile is a relative one, and the calculation of an absolute profile requires the knowledge of an extra constant—the absolute velocity at one point in depth. This value is not known in general unless the ship has an accurate navigation

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system and a 2-dimensional underwater log. The assignment of the resulting water velocity at the depth of the log ties down the velocity profile. There remains some uncertainty in the accuracy of the velocity profile at depth since errors in the measurement of shear could creep up additively in the integration procedure.

#### Conclusion

We have outlined a new concept for measuring the Richardson number profile and the velocity profile from survey vessels and have identified the potential problems in its implementation. The concept of measuring fluid vorticity appears to be feasible, although solid estimates of instrument and platform related noise sources are not yet available. The proof of concept rests on a follow up engineering design study that would emphasize the acquisition of hard numbers for the noise sources so as to optimize the final design options.

#### REFERENCES

- Bobrik, M. J. A simple method for estimating vortex intensity. J. AeroSpace Sci. 27, 957 (1960).
- Neil Brown, Instrument Systems, Inc. Acoustic Vector Averaging Current Meter (ACM-1), 1978.
- Garrett, C. and W. Munk. Space-time scales of internal waves: a progress report. J. Geophys. Res. 80, 291-297 (1975).
- Gibson, C. H. and R. B. Williams. Phase I—Engrg. Design Rept.—Development of a towed acoustic vorticity meter. Triadic Research, Inc., July, 1978.
- Gongwer, C. A. And E. Finkle, Turbulence meter: an instrument for oceanographic research. ISA Trans. 3, 217-225 (1962).

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- Gould, W. J., W. J. Schmitz, and C. Wunsch. Preliminary field results for a Mid-Ocean Dynamics Experiment (MODE-0). *Deep-Sea Res.* 21, 911-932 (1974).
- Govinda Raju, S. P., V. S. Holla, A. Nigam, and M. P. Verghese, Development of a vorticity meter. *J. Aircraft.* 5, 799-800 (1978).
- Hopkins, E. J. and N. E. Sorensen, A device for vortex core measurements. *J. Aeronaut. Sci.* 23, 396 (1956).
- Kovaszny, L. S. G., Turbulence measurements, Section F in *Physical Measurements in Gas Dynamics and Combustion* (ed. by R. W. Ladenburg, B. Lewis, R. N. Pease, and H. S. Taylor) Vol. 9 of High Speed Aerodynamics and Jet Propulsion Series, Princeton U. P., 227-228 (1954).
- Lakshminarayana, B. A simple device for the quantitative measurement of vortices. *AIAA Journal* 2, 1353 (1964).
- Osborne, T. R. Vertical profiling of velocity microstructure. *J. Phys. Ocean.* 4, 109-115 (1974).
- Rossby, H. T. A vertical profile of currents near Plantagenet Bank. *Deep-Sea Res.* 16, 377-385 (1969).
- Rossby, T. An oceanic vorticity meter. *J. Marine Res.* 33, 213-222 (1975).
- Sanford, T. B. Observation of the vertical structure of internal waves. *J. Geophys. Res.* 80, 3861-3871 (1975).
- Simpson, J. H. Observations of small scale vertical shear in the ocean. *Deep-Sea Res.* 22, 619-627 (1975).
- Todd, K. W. Some developments in instrumentation in air flow analysis. *Proc. Inst. Mech. Engrs* (London) 161, 213 (1949).



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Van Leer, J., W. Düing, R. Erath, E. Kennelly, and A. Speidel. The cyclesond: an unattended vertical profiler for scalar and vector quantities in the upper ocean. Deep-Sea Res. 21, 385-400 (1974).

Wigeland, R. A., M. Ahmed, and H. M. Nagib, Vorticity measurements using calibrated vane-vorticity indicators and cross-wires. AIAA Journal 16, 1229-1234 (1978).

Zalay, A. D. Hot-wire and vorticity meter wake vortex surveys. AIAA Journal 14, 694 (1976).

## Appendix

### ASSUMPTIONS IN DERIVATION OF EQUATION (12)

The equation governing the relationship between fluid vorticity and the sensor parameters is derived on pgs. 8-9. This appendix gives more detail on the assumptions implied in that derivation.

The travel time of an acoustic pulse or a constant phase point of a CW signal over a path of length  $L$  is

$$T = \frac{L^2}{\int_0^L (c + \mathbf{u} \cdot \mathbf{l}) \cdot d\mathbf{l}}, \quad (\text{A.1})$$

where  $c$  is the sound speed,  $\mathbf{u}$  is the fluid velocity vector, and  $\mathbf{l}$  is a unit vector tangent to the path. Assuming that the sound speed variation around the array is small (but possibly the sound speed is unknown), the equation simplifies to

$$T = \frac{L^2}{cL + \int_0^L \mathbf{u} \cdot d\mathbf{l}}. \quad (\text{A.2})$$

Denoting the travel time one way around the path as  $T_1$ ,

$$T_1 = \frac{L^2}{cL + \int_0^L \mathbf{u} \cdot d\mathbf{l}}, \quad (\text{A.3})$$

the travel time  $T_2$  the other way around the path becomes

$$T_2 = \frac{L^2}{cL - \int_0^L \mathbf{u} \cdot d\mathbf{l}}. \quad (\text{A.4})$$

The change in the sign of the integral in the denominator occurs because of the change in direction of the vector  $\mathbf{l}$ . The difference in these times is

$$\Delta T = T_2 - T_1 = \frac{2L^2\Gamma}{c^2L^2 - \Gamma^2} \quad (\text{A.5})$$

and the sum is

$$\bar{T} = T_2 + T_1 = \frac{2L^2(cL)}{c^2L^2 - \Gamma^2}, \quad (\text{A.6})$$

where  $\Gamma = \int_0^L \mathbf{u} \cdot d\mathbf{l}$ . Inspecting these two equations, it follows immediately that Eq. (11) results from Eq. (A.5) with the assumption that  $\Gamma \ll cL$ . Also, under the same assumption that  $\Gamma \ll cL$ , it is clear that the sum of the travel times is simply related to the average sound speed on the acoustic path. Thus, assuming that both  $\Gamma$  and the sound speed are unknown, and that  $\Delta T$  and  $\bar{T}$  are measured quantities, Eq. (A.5) and (A.6) constitute two nonlinear equations for two unknowns. These equations have a symmetry which permits the simple solution

$$\Gamma = \frac{2L^2 \Delta T}{\bar{T}^2 - \Delta T^2} \quad (\text{A.7})$$

$$cL = \frac{2L^2 \bar{T}}{\bar{T}^2 - \Delta T^2}. \quad (\text{A.8})$$

Thus, both  $c$  and  $\Gamma$  can be determined explicitly from the sum and difference of travel times. The ratio  $\Delta T/\bar{T}$  is the order of  $|\mathbf{u}|/c$  which is less than 0.01 for realistic values in the ocean, so that

$$\begin{aligned} \bar{T}^2 - \Delta T^2 &= \bar{T}^2 (1 - \Delta T^2/\bar{T}^2) \\ &\approx \bar{T}^2 \end{aligned}$$

to a very good approximation. In this case, Eqs. (A.7) and (A.8) reduce to the simpler forms

$$\Gamma = 2L^2 \Delta T / \bar{T}^2 \quad (\text{A.9})$$

$$c = 2L / \bar{T}. \quad (\text{A.10})$$

It is worth noting that the assumption that the sound speed varies only a small amount along the acoustic path is not necessary, as carrying the term  $\int_0^L c dl$  instead of simplifying it to  $cL$  throughout the analysis does not change the basic results. In that case, the only change in the results is that Eq. (A.10) is replaced by

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$$\int_0^L c dl = 2L^2/\bar{T}. \quad (\text{A.11})$$

Equation (12) follows from Eq. (A.9) upon substitution of expression (8) in the latter, and this exercise has underscored the accuracy of the assumptions which are implied in Eq. (12). This derivation also has made it clear that the instrument does not require a separate measurement of sound speed as a simple application of Eq. (12) would imply.